Reg. No. : [| | | | | | | | | | |]

Question Paper Code : 70766

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to All Branches)

(Regulations 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Prove that $3x^2y\vec{i} + (yz - 3xy^2)\vec{j} - \frac{z^2}{2}\vec{k}$ is a solenoidal vector.

2. State Green's theorem.

3. Find the particular integral of $(D^2 + 2D + 1)y = e^{-x}x^2$.

4. Convert the equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \log x$ into a differential equation with constant coefficients.

5. State sufficient condition for the existence of Laplace transform.

6. Find the inverse Laplace transform of $\frac{s^2 - 3s + 2}{s^3}$.

7. Show that $|z|^2$ is not analytic at any point.

8. Find the invariant points of the transformation $w = \frac{z-1}{z+1}$.

9. Define and give an example of essential singular points.

10. Express $\int_0^{\pi} \frac{d\theta}{2\cos\theta + \sin\theta}$ as complex integration.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the directional derivative of $4x^2z + xy^2z$ at $(1, -1, -2)$ in the direction of $2\vec{i} + \vec{j} - 3\vec{k}$. (6)

(ii) Using Stoke's theorem evaluate $\iint_S curl\vec{f} \cdot \vec{n}\, ds$ given $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$. (10)

Or

(b) (i) Find ∇r^n and hence prove that $\nabla^2 r^n = n(n+1)r^{n-2}$. (6)

(ii) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1, z = 0$ and $z = 1$.

12. (a) (i) Solve : $(D^2 + 5D + 4)y = e^{-x} \sin 2x + 2e^{-x}$. (8)

(ii) Solve the differential equation $(D^2 + 4)y = \sec^2 2x$ by the method of variation of parameters. (8)

Or

(b) (i) Solve : $(1+x)^2 \dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 2\sin[\log(1+x)]$. (8)

(ii) Solve : $(D+2)x + 3y = 2e^{2t}$; $3x + (D+2)y = 0$. (8)

13. (a) (i) Find the Laplace transform of the following functions : (8)

(1) $\dfrac{e^{-t} \sin t}{t}$

(2) $t^2 \cos t$.

(ii) Using Laplace transform, solve $(D^2 + 3D + 2)y = e^{-3t}$ given $y(0) = 1$ and $y'(0) = -1$. (8)

Or

(b) (i) Using convolution theorem, find $L^{-1}\left\{\frac{s}{(s^2+4)(s^2+9)}\right\}$. (8)

(ii) Find the Laplace transform of the square wave function defined by

$$f(t)=\begin{cases} k, & 0<t<\frac{a}{2},\ f(t+a)=f(t) \\ -k, & \frac{a}{2}<t<a \end{cases}.$$ (8)

14. (a) (i) Determine the analytic function $w=u+iv$ if $u=e^{2x}(x\cos 2y-y\sin 2y)$. (8)

(ii) Show that a harmonic function 'u' satisfies the formal differential equation $\frac{\partial^2 u}{\partial z\,\partial\bar{z}}=0$ and hence prove that $\log|f'(z)|$ is harmonic, where $f(z)$ is a regular function. (8)

Or

(b) (i) Find the image in the w-plane of the infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$. (8)

(ii) Find the bilinear transformation that maps the points $z=0,-1,i$ into the points $w=i,0,\infty$ respectively. (8)

15. (a) (i) Using Cauchy's integral formula evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}\,dz$ where C is $|z|=2$. (4)

(ii) Evaluate $\int_0^\infty \frac{dx}{x^4+a^4}$ using contour integration. (12)

Or

(b) (i) Obtain the Laurent's expansion of $f(z)=\frac{z^2-4z+2}{z^3-2z^2-5z+6}$ in $3<|z+2|<5$. (6)

(ii) Evaluate $\int_C \frac{z^3\,dz}{(z-1)^4(z-2)(z-3)}$ where C is $|z|=2.5$ using residue theorem. (10)

———————